Introduction

Optical frequency combs, originally generated by ultrafast mode-locked lasers, have been successively demonstrated in continuously-pumped Kerr microresonators [1], and in quantum cascade lasers [2]. Materials with second-order susceptibility, $\chi^{(2)}$, are routinely used for transferring otherwise generated OFCs to different spectral regions. Recently, we have observed and theoretically explained frequency comb generation in a continuously-pumped cavity-enhanced second-harmonic generation (SHG) system, where multiple, cascaded $\chi^{(2)}$ nonlinear processes enable the onset of broadband $\chi^{(2)}$-comb emission, both around the fundamental pump frequency and its second harmonic [3]. Our system shows a remarkable similarity with generation and dynamics of frequency combs in Kerr microresonators, as confirmed by a simple three-wave theoretical model we developed.

The experiment

The system is based on a periodically-poled LiNbO$_3$ crystal placed in a travelling-wave optical cavity and pumped by an amplified cw Nd:YAG laser. When the crystal is phase-matched for SHG of the resonant fundamental frequency and the laser power exceeds a threshold value, the second-harmonic power clamps at a constant value and a signal/idler pair starts to oscillate, frequency symmetric around the fundamental frequency. Increasing the pump power above the OPO threshold, more sidebands appear around the fundamental mode, forming a comb whose teeth are equally spaced by an integer multiple $\Delta \nu$ of the cavity free spectral range (FSR). Further increasing the pump power secondary parametric oscillations occur, resulting in the appearance of secondary frequency combs around the primary comb teeth, similarly to the hierarchical comb formation in Kerr microresonators [4,5].

Increasing the crystal temperature, the original SHG process becomes positive phase mismatched and the off-phase matched pump frequency acts as a seed for a 1-FSR-spaced comb, similarly to a primary sidemode in the previous case of quasi-phase-matched SHG (Fig. 1a). The appearance of comb emission is confirmed by detecting the intermodal beat notes at

![Fig. 1](image-url)

Fig. 1 (a) Calibrated optical spectrum of the IR cavity output for phase-mismatched SHG. (b) detail of the RF spectrum of the IR emission with the intermodal beat note at the FSR frequency of 493 MHz.
multiple of the FSR for both the infrared and visible combs (Fig. 1b).

The theoretical model

The simplest nontrivial set of equations includes at least a pair of parametric fields in addition to the fundamental pump field. If we consider a set of coupled mode equations for the second-order interaction processes, taking into account second harmonics and sum frequencies as well, a perturbative solution leads to a set of dynamic equations for the three sub-harmonic fields [3]:

\[ \dot{A}_0 = -(\gamma + i\Delta_0) A_0 - 2g_0\eta_0 s A_0^* A_s A_i - g_0(\eta_{00}|A_0|^2 + 2\eta_{0s}|A_s|^2 + \eta_{0i}|A_i|^2)A_0 + F_{in} \]  
(1a)

\[ \dot{A}_s = -(\gamma + i\Delta_s) A_s - g_0\eta_{00 s} A_0^2 A_i^* - g_0(2\eta_{s0}|A_0|^2 + \eta_{ss}|A_s|^2 + 2\eta_{si}|A_i|^2)A_s \]  
(1b)

\[ \dot{A}_i = -(\gamma + i\Delta_i) A_i - g_0\eta_{00 i} A_0^2 A_s^* - g_0(2\eta_{i0}|A_0|^2 + 2\eta_{is}|A_i|^2 + \eta_{ii}|A_i|^2)A_i \]  
(1c)

where subscripts 0, ‘s’ and ‘i’ indicate fundamental, signal and idler modes, respectively. The \( A \)'s are the normalized electric field amplitudes; \( F_{in} \) is the pump amplitude coupled into the cavity; \( \gamma \) is the cavity decay constant, assumed to be the same for the three fields; the \( \Delta \)'s are the cavity detunings of the respective modes; the \( \eta \)'s are effective third-order complex susceptibilities, depending on the wave-vector mismatches of the considered second-order processes; and \( g_0 \) is a common gain factor depending on the crystal length and the second-order coupling strength.

Eqs. (1) describe the elemental dynamics of the cavity SHG-OPO system in terms of effective third-order interactions between the three sub-harmonic fields [6]. The dynamics of the harmonic fields is “slaved” to the sub-harmonic fields, i.e. their amplitudes instantaneously follow the sub-harmonic amplitudes, on the time scale of the cavity round-trip time. The harmonic fields, however, physically mediate the effective  interaction of Eqs. (1), eventually leading to comb formation in both spectral ranges.

\( \chi^{(2)} \)-combs have several advantages with respect to Kerr-combs based on \( \chi^{(3)} \) materials, exploiting the intrinsically higher efficiency of \( \chi^{(3)} \) processes, moreover they can be realized all over the transparency range of the nonlinear material without the more severe constraint on the dispersive characteristics of \( \chi^{(3)} \)-based devices, while the simultaneous occurrence of octave-distant combs provides a useful metrological link between two spectral regions without the need for a full octave-wide comb. The analogy with Kerr-combs unveils the possibility to predict and observe most of the effects occurring in Kerr cavities, paving the way for a novel class of highly efficient and versatile frequency comb synthesizers based on second-order nonlinear materials.

References


